1. The variables x and y satisfy the relation $\sin y = \tan x$, where $-\frac{1}{2}\pi < y < \frac{1}{2}\pi$. Show that [5]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\cos x\sqrt{\cos 2x}}.$$

$\cos y \cdot \frac{dy}{dx} = \sec^2 x$ ⟵ B1

$\frac{dy}{dx} = \frac{\sec^2 x}{\cos y}$ ⟵ B1

$= \frac{1}{\cos y \cos^2 x}$

$= \frac{1}{\sqrt{1-\tan^2 x} \cdot \cos^2 x}$ ⟵ B1 in terms of x

$= \frac{1}{\sqrt{\cos^2 x - \sin^2 x} \cdot \cos x}$ ⟵ M1 double angle

$= \frac{1}{\sqrt{\cos 2x} \cdot \cos x}$ — A1

2. The diagram shows the curve with equation

$$x^3 + xy^2 + ay^2 - 3ax^2 = 0,$$

where a is a positive constant. The maximum point on the curve is M. Find the x–coordinate of M in terms of a. [6]

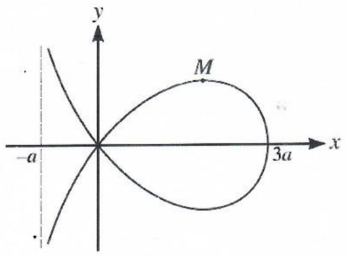

Figure 1: Curve

B1 B1

$3x^2 + y^2 + x \cdot 2y\frac{dy}{dx} + a \cdot 2y\frac{dy}{dx} - 6ax = 0$

$\frac{dy}{dx} = \frac{3x^2 + y^2 - 6ax}{-(2xy + 2ay)}$ M1

$3x^2 + y^2 = 6ax$ or $y^2 = 6ax - 3x^2$ A1

$x^3 + (x+a)(6ax - 3x^2) - 3ax^2 = 0.$

$x^3 + (6ax^2 - 3x^3 + 6a^2x - 3ax^2) - 3ax^2 = 0$

M1

$-2x^3 + 6a^2x = 0$

$x(6a^2 - 2x^2) = 0$

$\Rightarrow x^2 = 3a^2$ $x = \sqrt{3}a$ A1

3. The parametric equations of a curve are

$$x = \ln(\tan t), \qquad y = \sin^2 t,$$

where $0 < t < \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of t. [4]

(ii) Find the equation of the tangent to the curve at the point where $x = 0$. [3]

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\sin t \cos t}{\frac{\sec^2 t}{\tan t}}$$ B1 B1 M1

$$= 2\sin t \cos t \tan t \cdot \cos^2 t$$

$$= 2\sin^2 t \cos^2 t$$ A1

$x = 0$, $\tan t = 1$ $\quad t = \frac{\pi}{4}$ B1

$\sin t = \frac{\sqrt{2}}{2}$ $\quad \sin^2 t = \cos^2 t = \frac{1}{2}$

$\Rightarrow \frac{dy}{dx} = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$. M1

$y - \frac{1}{2} = \frac{1}{2}(x - 0)$ A1

4. The diagram shows the curve $y = 10e^{-\frac{1}{2}x}\sin 4x$ for $x \geq 0$. The stationary points are labelled T_1, T_2, T_3, $\cdots$ as shown.

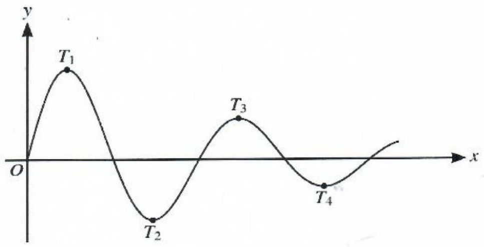

Figure 2: Curve

(i) Find the x-coordinates of T_1 and T_2, giving each $x-$coordinate correct to 3 decimal places. [6]

(ii) It is given that the $x-$coordinate of T_n is greater than 25. Find the least possible value of n. [4]

$\frac{dy}{dx} = 10\left(-\frac{1}{2}e^{-\frac{1}{2}x}\sin 4x + e^{-\frac{1}{2}x} \cdot 4\cos 4x\right)$ M1

A1

$= -5e^{-\frac{1}{2}x}\left(\sin 4x - 8\cos 4x\right)$

$\Rightarrow \tan 4x = 8$ M1 A1

$4x = \tan^{-1}(8) + k\cdot\pi$. A1

$x = \frac{\tan^{-1}(8)}{4} + \frac{1}{4}k\pi$. A1

(i) $T_1 = \cancel{20.77^\circ}\ 0.362$, $T_2 = 1.147$

(ii) $0.362 + \frac{\pi}{4}\cdot(n-1) > 25$ B1 M1

$n > 32.3$ $n = 33$ A1 n.

A1

32.33